Covariance Manipulation for Conjunction Assessment

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Introduction

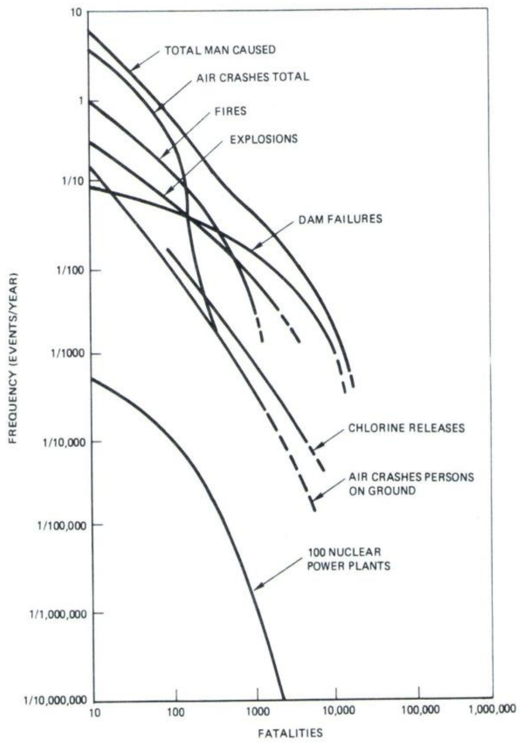
- **Use of probability of collision (Pc) has brought sophistication to CA**
  - Made possible by JSpOC precision catalogue because provides covariance
  - Has essentially replaced miss distance as basic CA parameter
- **Embrace of Pc has elevated methods to "manipulate" covariance to enable/improve CA calculations**
- **Two such methods to be examined here**
  - Compensation for absent or unreliable covariances through "Maximum Pc" calculation constructs
  - Projection (not propagation) of epoch covariances forward in time to try to enable better risk assessments
- **Two questions to be answered about each**
  - Situations to which such approaches are properly applicable
  - Amount of utility that such methods offer

Agenda

- **Risk assessment fundamental principles, following Kaplan**
- **Absolute and relative Maximum Pc methods explained**
  - Unconstrained/unknown primary and secondary covariances
  - One covariance constrained/known
- **Absolute and relative Maximum Pc methods evaluated**
  - Theoretical/philosophical limits
  - Practical limits
- **Epoch covariance projection methods described**
- **Epoch covariance projection methods evaluated**
  - For assessing benefit of additional tracking data
  - For CA risk assessment
    - Projection to time of closest approach (TCA)
    - Projection to actual remediation decision point
- **Conclusions**

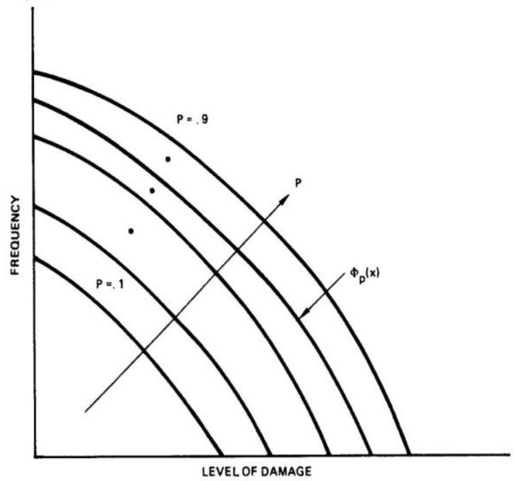
Kaplan Triplet

- **Idea is that risk is combination of likelihood and consequence**
  - Kaplan "triplet" term arises from enumerating risk scenarios S_i, each of which has probability P_i and consequence X_i:
    - $< S_1, P_1, X_1 >$, $< S_2, P_2, X_2 >$, &c.
  - Plotting all of the (P_i, X_i) ordered pairs produces a risk curve (example at left)
- **Appears commonly in risk management constructs**
  - Risk is combination of likelihood and consequence
  - Sometimes frames as *product* of likelihood and consequence, but this assumes risk neutrality



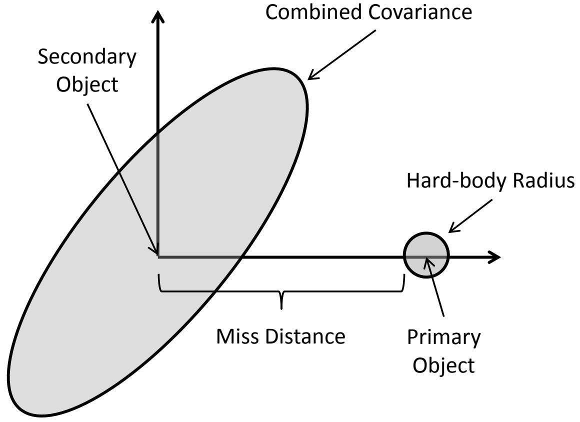
Uncertainty of Likelihood

- **Each Kaplan triplet is a point estimate of likelihood and consequence arising from a particular scenario**
- **However, uncertainty actually exists in both estimates**
  - Assessing likelihood of an event is process in which inputs contain error, giving the calculation an uncertainty
  - If likelihood is of a particular events taking place, there is uncertainty in predicting the consequences of that event
    - Could set the likelihood portion to be the likelihood of certain consequences rather than a given scenario, but this violates the framework of the Kaplan Triplet
- **Risk “curves” are thus actually a family of curves, or probability density**
  - One chooses the risk curve to match desired level of certainty in estimate of likelihood and consequence
  - Example from Kaplan article at right



Maximum Pc ("PcMax") Constructions: Both Covariances Unconstrained

- **Canonical conjunction plane plot at right**
  - Ellipse at origin: projected sum of primary and secondary covariances
  - Circle on x-axis: projected sphere that represents adjoined sizes of primary and secondary objects
  - Pc: portion of combined covariance probability density that falls within HBR area

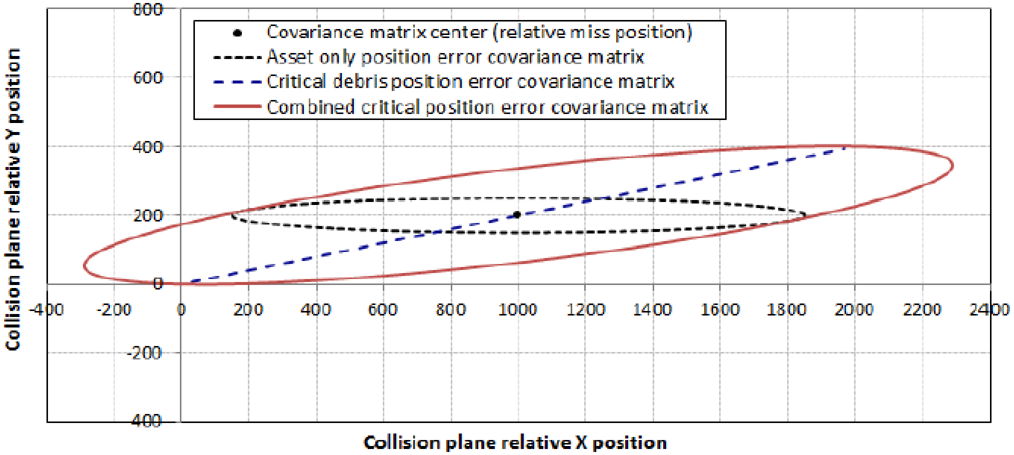


- **Pc governed by ratios among miss distance, HBR, and covariance size, aspect ratio, and orientation**
- **If covariance allowed to assume any size/shape/orientation, can develop expressions for conditions to produce maximum Pc**
  - Relationships worked out formally in important paper by Alfano*
  - Formulae for PcMax if conjunction plane aspect ratio is known
  - If covariance allowed to be essentially degenerate and lie along the miss vector, simple formula for Pc maximum value that is approached

*Alfano, S. "Relating Position Uncertainty to Maximum Conjunction Probability." *Journal of the Astronautical Sciences*, Vol. 53 No. 2 (April-June 2005), pp. 193-205.

Maximum Pc ("PcMax") Constructions: One Covariance Unconstrained

- **Frisbee* extended Alfano's work to consider cases in which one of the two satellite's covariances known and the other unconstrained**
- **Similar conceptually (note degenerate ellipse for debris object), but known covariance pulls probability density away from miss vector**
  - Pc is thus lower than unconstrained PcMax technique
- **Approximate solutions without, and more exact solutions with, numerical integration**



*Frisbee, J.H. "An Upper-Bound on High-Speed Satellite Collision Probability when only one Object has Position Uncertainty Information." AAS/AIAA Astrodynamics Specialist Conference (paper # 15-717), Vail CO, August 2015.

PcMax Philosophical Issue: PcMax Calculations are not True Pc Values

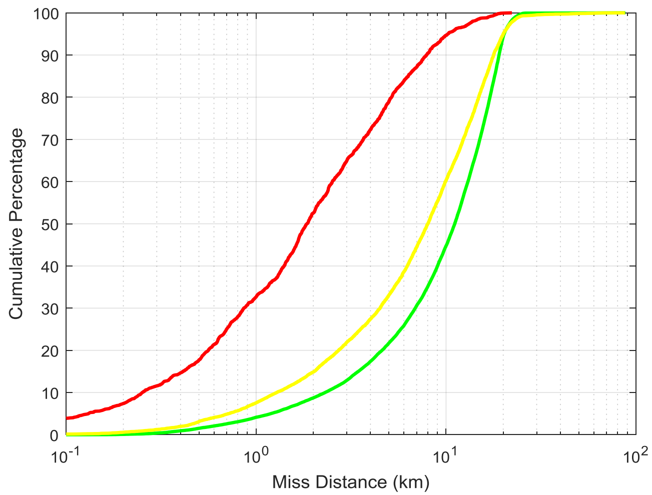
- **Kaplan Triplet: scenario, likelihood, and consequence**
- **Regular Pc values give likelihood of collision**
  - Have legitimate role in Kaplan Triplet and thus risk assessment
- **MaxPc values do not give a likelihood**
  - Indicate a maximum possible Pc value should certain conditions inhere
    - Actually more like a consequence than a likelihood
  - To use in risk assessment context, would need to multiply PcMax by a probability that the conditions that produce the PcMax will arise
  - Since PcMax used because one or both covariances absent or untrustworthy, unlikely that a probability can be assigned to these conditions' arising
- **Neither Alfano nor Frisbee recommends PcMax as a direct risk assessment parameter**
  - However, this is suggested and attempted by some practitioners
- **May have some utility as pre-filter**
  - Investigation follows

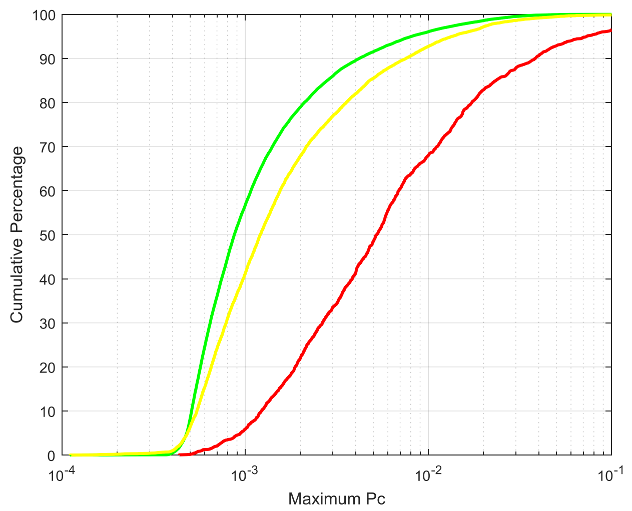
Practical Utility of PcMax Constructs: Evaluation Dataset

- **To determine actual utility of PcMax parameters, should evaluate against conjunction database:**
  - Conjunction history for 11 NASA CA protected satellites (all ~700km orbits)
  - May 2015 to May 2016
  - Screening volume of 0.5 km x 17 km x 20 km (RIC)
- **Helpful to separate events/reports by level of severity; CARA uses color scheme**
  - Green: not worrisome; Pc < 1E-07
  - Red: worrisome; Pc < ~1E-04 - 5E-04; 1E-04 used in this analysis
  - Yellow: not worrisome but has potential to become so; between green and red
- **Dataset contains ~72,000 conjunction reports**
  - 85% green, 12.2% yellow, 2.8% red

Practical Utility of PcMax for Present Risk: Unconstrained PcMax

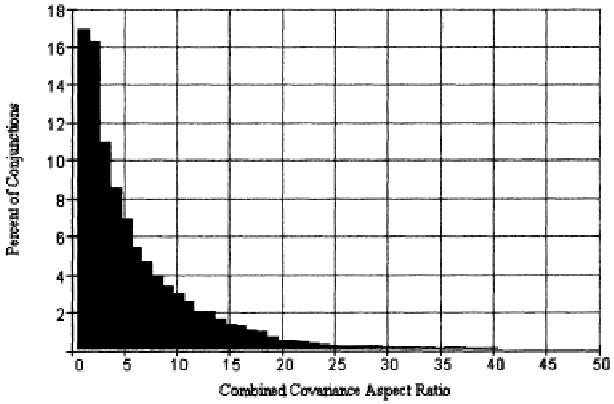
- **Unconstrained PcMax function only of HBR and miss distance (MD)**
  - With fixed HBR (20 m here), function only of miss distance
- **With 0.5 x 17 x 20 km screening volume, MDs constrained**
  - Will impute a lower bound to PcMax
- **Graphs show MD (left) and PcMax (right) for evaluation dataset**
- **Essentially no PcMax values below red threshold (1E-04)**
  - Unconstrained PcMax not very useful as pre-filter

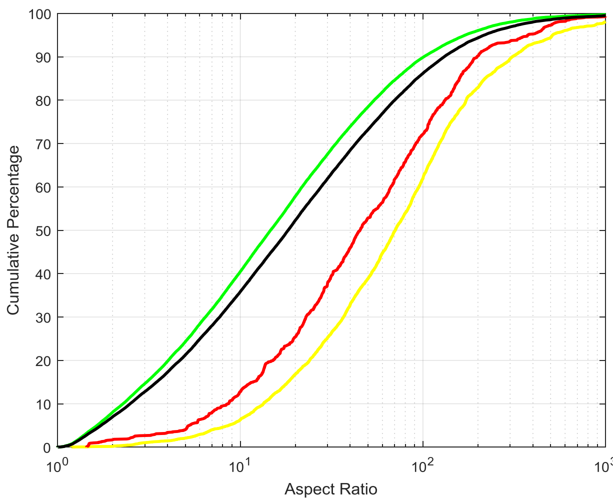




Practical Utility of PcMax for Present Risk: PcMax Constrained by Aspect Ratio

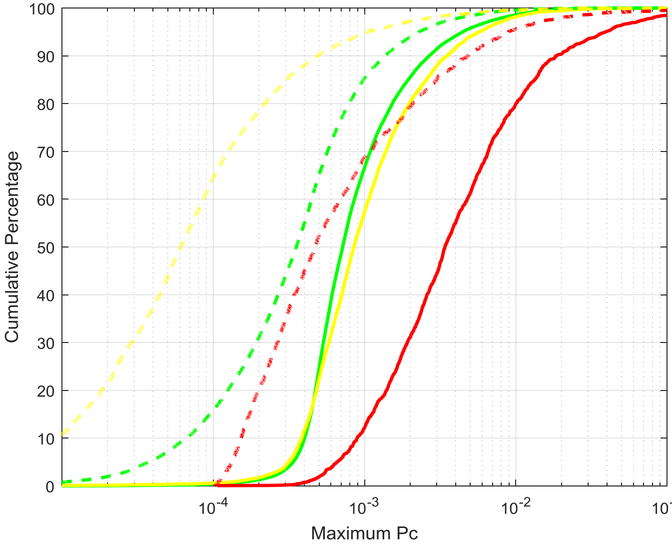
- **Alfano provides formulae for PcMax calculations when conjunction plane combined covariance aspect ratio (AR) is known**
  - Provides more realistic/bounded result than global PcMax
  - Approximation formulae tested for ARs 1-50, with good results
  - Profiling of 2004 catalogue given in left graph below; almost no ARs > 50
- **Similar profiling of current examination dataset (2015-16) secondaries, given in graph at right**
  - Much larger range of ARs, and much larger values
  - More difficult to claim that could estimate AR adequately





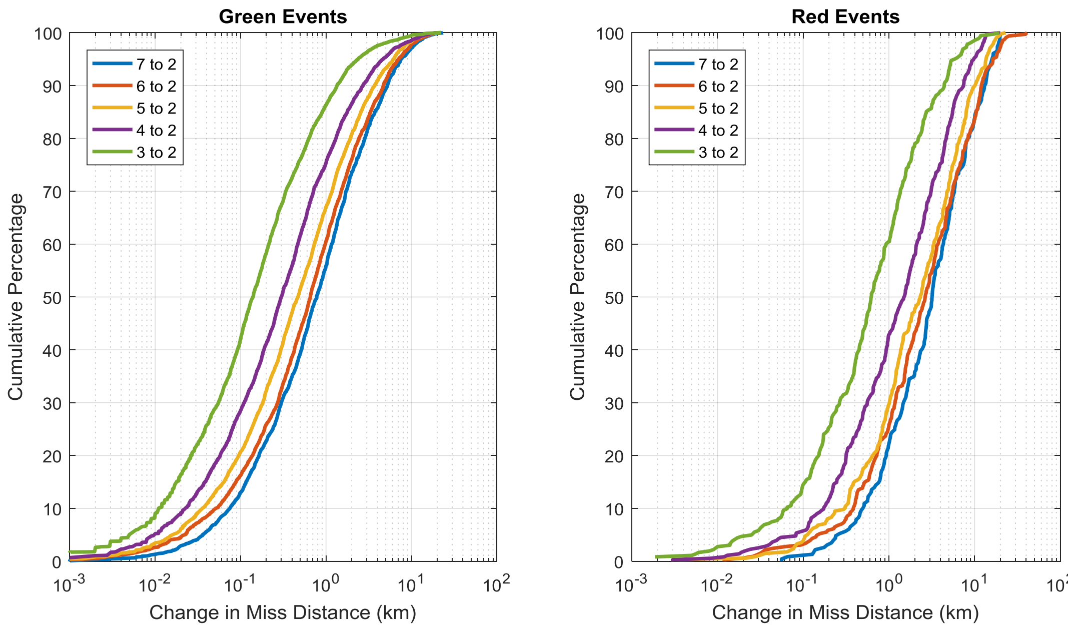
Practical Utility of PcMax for Present Risk: PcMax Constrained by One Covariance

- **Two common scenarios for Frisbee technique**
  - Primary covariance known (because O/O produces it) but have only GP catalogue for secondary, so secondary covariance unknown
  - Secondary covariance known (from JSpOC precision catalogue) but primary covariance unknown because not produced by O/O and satellite is maneuvring
- **Results from examination catalogue profiling below**
  - Solid lines: secondary covariance unknown; dashed lines: primary unknown
  - Secondary unknown not very promising; primary unknown has some utility



Practical Utility of PcMax for Future Risk

- **Can a PcMax technique bound the Pc values of future updates?**
- **Success of this application requires unchanged nominal MD**
  - Techniques determine maximum Pc for a certain nominal MD; to be predictive, this MD must endure throughout entire event
- **Graphs below show MD history for examination dataset**
  - Too much variation—especially for red events—to claim PcMax is predictive



PcMax Techniques: Conclusion

- **Theory behind these techniques certainly sound**
- **Authors were modest about their applications**
- **Not directly usable as risk assessment parameters**
- **Not particularly effective as pre-screening filter**
  - With modern precision catalogue and typical NE screening volume sizes, not particularly effective as pre-filter for current risk
  - Given historically-observed changes in precision catalogue nominal miss values as events develop, not particularly effective as bounding function on future risk
- **Can have an operational role, but probably only in extreme situations**

Covariance Projection: Agenda

- **JSpOC covariance formation basics**
- **Covariance propagation vs covariance projection**
- **Applications of covariance projection**
  - For CA risk assessment
    - Projection to time of closest approach (TCA)
    - Projection to actual remediation decision point
  - For assessing benefit of additional tracking data

Batch Epoch Covariance Generation (1 of 2)

- **Batch minimum variance update (ASW method) uses the following minimization equation**
  - $dx = (A^T W A)^{-1} A^T W b$
    - dx is the vector of corrections to the state estimate
    - A is the time-enabled partial derivative matrix, used to map the residuals into state-space
    - W is the "weighting" matrix that provides relative weights of observation quality (usually 1/σ, where σ is the standard deviation generated by the sensor calibration process)
    - b is the vector of residuals (observations – predictions from existing state estimate)
- **Covariance is the collected term $(A^T W A)^{-1}$**
  - A the product of two partial derivative matrices:
    - $A = \frac{\partial(obs)}{\partial X_0} = \frac{\partial(obs)}{\partial X}\frac{\partial X}{\partial X_0}$
    - First term: partial derivatives of observations with respect to state at obs time
    - Second term: partial derivatives of state at obs time with respect to epoch state

Batch Epoch Covariance Generation (2 of 2)

- **Formulated this way, this covariance matrix is called an *a priori* covariance**
  - A does not contain actual residuals, only transformational partial derivatives
  - So $(A^TWA)^{-1}$ is a function only of the amount of tracking, times of tracks, and sensor calibration relative weights among those tracks
    - Not a function of the actual residuals from the correction
  - Not an actual statement of fit error, but an estimate of expected fit error
- **Allows for estimation of epoch covariance without requiring actual observational measurements**
  - If tracking details can be projected, then so can epoch covariance
  - Usual techniques can be used to propagate projected covariance to time-points of interest
- **Question: what are legitimate and proper uses of a projected covariance?**

Projected Covariances: To Future Decision Point

- **Suppose one is six days from TCA; will make remediation decision at 2 days from TCA**
  - Current covariance will need to be propagated ~6 days; will become large
- **Could "project" a covariance to 2 days from TCA (decision point)**
  - Will need to predict the expected tracking times, amounts, and stations
  - Projected covariance will need only 2 days' propagation; will be smaller
- **Should give a reasonable guess of the situation that will be encountered when a decision is to be rendered**
- **Can provide a basis for determining if additional data helpful**
  - Can examine projected case with and without increased tracking
  - If increased tracking changes calculated Pc appreciably, then have firm justification for requesting it
    - Some conjunctions relatively insensitive to increased tracking; this would reveal that
- **Reasonable use of projected covariance**

Projected Covariances to Future Decision Point: Limitations

- **Presumes nominal miss distance and conjunction geometry remain static over a number of days**
  - Shown earlier to be questionable, even fallacious assumption
- **Requires ability to predict future tracking accurately**
  - The amounts, times, and sources of predicted future tracking
    - Will show some data on this presently
  - The data that will be excluded from future ODs
    - Dynamic LUPI algorithm and manual exclusions
- **A guess at the situation at 2 days to TCA, but not a substitute for it**
  - Can only make decision based on the data you actually have received
  - Otherwise, why even ask for increased tasking—you believe you know what will happen if you do, so why even bother to get it?
  - Otherwise, why not postulate an essentially infinite amount of extremely precise tracking, which will push the risk to 0 if MD > HBR?
- **So need to wait for 2-day point to see what is actually obtained**

Projected Covariances: To TCA (1 of 2)

- **Projecting covariances to TCA seems *prima facie* like a good idea**
  - No covariance propagation error—essentially an epoch covariance at TCA
  - Most important point in the event development—would want to know the conditions at that point more than any other
- **However, compare to 7-to-2 projection case**
  - In 7-to-2 case, opportunity exists to receive actual data to ground decision
    - Projection is really useful only to determine if tracking increases would be helpful
  - In projection to TCA case, there is no ability to receive the actual projected tracking data
  - If not willing to make decision at 7 days based on projection to 2 days, then should not be willing ever to make decision based on projection to TCA

Projected Covariances: To TCA (2 of 2)

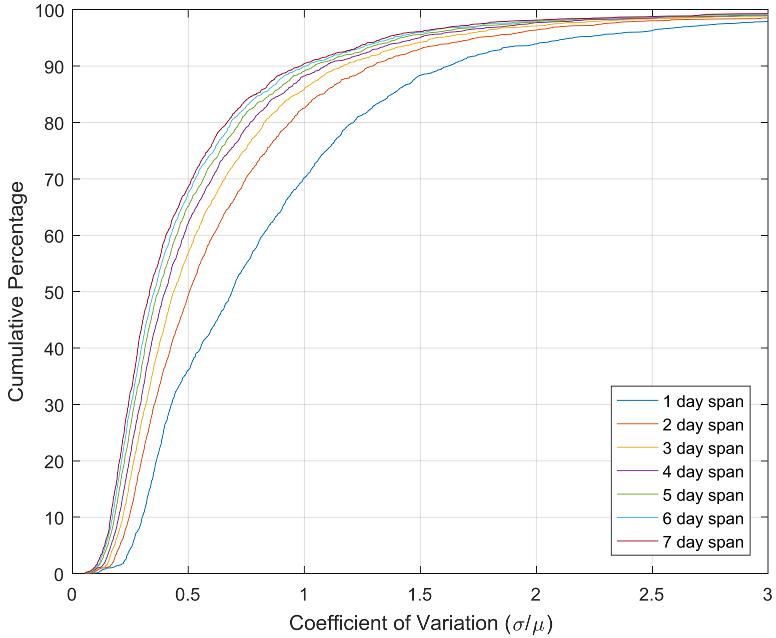
- **Thought experiment: risk as function of projected tracking**
  - At 2 days to TCA; Pc from nominal covariance is 5E-05; projected covariance to TCA give Pc of 5E-04
    - Should satellite maneuver based on projection?
  - JSpOC calls—secondary chosen for experimental satellite for pursuing highest level of JSpOC tracking with most accurate sensors
    - Projected covariance re-run with this new tracking level, and Pc from that now 1E-06
    - Should planned maneuver now be cancelled?
  - Now JSpOC calls and projects bevy of sensor outages—no tracking before TCA likely
    - Back to original Pc value of 5E-05
- **What the sensor network may or may not do after the maneuver decision point cannot have any effect on the conjunction risk at the time of decision**
  - Risk assessments can be made only on the basis of actual tracking received, not fictional data that might or might not be received after the decision point

Projected Covariances: Projection Uncertainties

- **Disagreements about utility of construct for risk assessment**
- **Agreement that can be useful to determine if additional data helpful**
- **However, utility substantially affected by ability to predict future tracking levels**
  - How stable and reliable are secondary object tracking levels?
- **Full evaluation of this question complex**
  - Must examine quiescent-level tracking response, higher-category tracking response, and consistency of tracking rates from search-based sensors
- **However, can conduct abbreviated investigation to assemble first-order answer**

Projected Covariances: Projection Uncertainties in Tracking Levels

- **Tracking levels for ~2100 secondaries in evaluation dataset profiled over one-year period**
  - Year divided into time-spans of 1 day to 7 days
  - Mean and standard deviation of # of tracks in each bin size computed
  - Coefficient of variation (σ/μ) calculated for each object for each span; CDF graph given below



- **Values > ~0.5 yield difficult prediction situation**
  - If mean = 20 tracks/day, then CoV of 0.5 means 32% < 10 or > 30 tracks/day—large difference
  - 40-50% of cases have CoV > 0.5; 10-15% have CoV > 1
- **Predicting tracking levels reliably will be very difficult**
  - Viable perhaps only with PDF of Pc values

Conclusions

- **Kaplan risk assessment components useful in determining utility of covariance manipulation techniques**
- **PcMax techniques**
  - Are not actual statements of likelihood and thus cannot have any direct risk assessment role
  - As pre-filters are not nearly as effective may have been with a GP catalogue
- **Covariance projection**
  - Can have a role in deciding whether additional data collection &c. can help
  - Cannot have a direct role in risk assessment decisions
    - Projections to decision point must be verified by actual receipt of data
    - Projections to TCA cannot be so verified and therefore are not helpful
  - More work needed on methods to assess covariance projection uncertainty
    - Not a simple task
    - Must not pursue a technique that merely makes covariances larger